

October 11 2013

Mat 3172

Professor M. Alvo

Time: 75 minutes

Name of Student:

Student Number:

This is an open book exam. Calculators allowed. Answer all the questions.

1. (4) a) If  $A, B$  and  $C$  are three independent events with  $P(A) = 0.5, P(B) = 0.2, P(C) = 0.3$ , find the probability that at least one occurs.

b) Let  $E$  and  $F$  be two disjoint events such as  $P(E) = 0.6$ , and  $P(F) = 0.15$ . Calculate  $P(E^c \cap F^c)$ .

2. (4) Show that for  $n \geq 1$ , events  $E_1, \dots, E_n$ , the following inequality holds:

$$P\left(\bigcap_{i=1}^n E_i\right) \geq \sum_{i=1}^n P(E_i) - (n-1)$$

3. (3) Suppose that cards from a deck of 52 cards are dealt one at a time. Given that the first ace is the 10<sup>th</sup> card dealt, what is the probability that the card following it is the ace of spades?

4. (3) Suppose that in the gambler's ruin problem you start with  $\$i$  but your opponent has an infinite amount of money. Let  $p$  be the probability of heads.

a) Show that if  $p \leq 0.5$ , you will go broke with probability 1.

b) Show that if  $p > 0.5$ , you will go broke with probability  $\left(\frac{1-p}{p}\right)^i$ .

5. (6) An urn contains 3 white and 2 black balls. Two balls are drawn without replacement.

a) Calculate the probability that the second ball is black given that the first is black.

b) Calculate the probability that the second ball is the same color as the first ball

c) Calculate the probability that the first ball is white given that the second ball is white